FUNDAMENTALS OF HEAT TRANSFER AND ITS SOLUTION USING THE FINITE ELEMENT METHOD

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1. Conduction and convection

1.1 Basic analytical equations

For the temperature function describing the non-stationary temperature field $T \equiv T(x,y,z,t)$ in a general thermally orthotropic body (Fig.1) the differential equation

$$\frac{\partial}{\partial x}\left(\lambda_x \frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(\lambda_y \frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(\lambda_z \frac{\partial T}{\partial z}\right)+Q=\rho c \frac{\partial T}{\partial t} \tag{1.1}$$

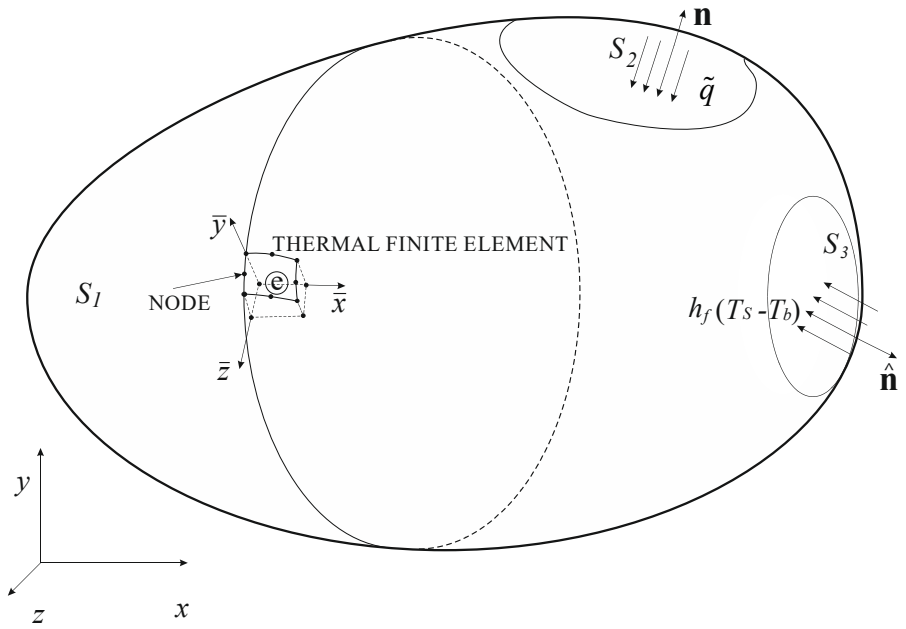


Fig.1 *General 3D body with temperature field*

holds, where:

ρ = density [kg/m^3]
c = specific heat capacity [J/(kgK)]
T = temperature [K]
t = time [s]
$\lambda_x, \lambda_y, \lambda_z$ = conductivity in the *x*, *y* and *z* directions [W/(mK)]
Q = heat generation rate per unit volume [W/m^3]

We consider a linear problem for which the following assumptions apply:

- Material parameters are constant - they do not depend on temperature or time
- There are no phase changes in the body
- Heat transfer by radiation is not considered

Uneven temperature distribution in the body will cause heat flux q [W/m^2]

$$q(x,y,z,t)=-\left(\lambda_x \frac{\partial T}{\partial x}+\lambda_y \frac{\partial T}{\partial y}+\lambda_z \frac{\partial T}{\partial z}\right) \tag{1.2}$$

which, after determining the function T, can be calculated from this relation.

Since (1.1) is a 1st order differential equation with respect to time, we need to give one *initial condition* for it

$$T\left(x,y,z,t=0\right)=T_0\left(x,y,z\right) \tag{1.3}$$

where the function T_0 indicates the temperature distribution at the beginning of the solution of the problem in the entire volume of the body.

In addition, for a 2nd order differential equation (with respect to positional variables), we need to specify two boundary conditions. Dirichlet's boundary condition in this case is

$$T\left(x,y,z,t\right)=T_1\left(x,y,z,t\right) \quad \text{on } S_1 \tag{1.4}$$

where S_1 is the part of the surface of the body where we have prescribed the temperature.

Cauchy's boundary condition prescribes heat flux $\tilde{q}$ (in W/m^2) across the surface S_2

$$\tilde{q}=\lambda_n\frac{\partial T}{\partial n}=\lambda_x\frac{\partial T}{\partial x}\cos\alpha+\lambda_y\frac{\partial T}{\partial y}\cos\beta+\lambda_z\frac{\partial T}{\partial z}\cos\gamma \quad \text{on} \quad S_2 \tag{1.5}$$

where the directional cosines of the external normal to the surface emerge.

FEM programs usually react to unspecified "mandatory" conditions as follows (it is useful to check this)

- An unspecified initial condition $\Rightarrow$ at all points of the solid is a temperature equal to zero at the beginning of the solution of the problem
- An unspecified boundary condition for temperature $\Rightarrow$ at all points on the surface of a solid a condition for heat flow holds.
- An unspecified boundary condition for heat flux $\Rightarrow$ at all points on the surface of the solid (where no boundary condition for temperature is specified) is the heat flux equal to zero – so the body is on the surface S_2 thermally insulated.

It also follows from the above that in the case of 2D problems the body is thermally insulated in the direction perpendicular to the plane of the solution.

For heat conduction problems, in addition to the "mandatory" boundary conditions, additional conditions can be specified in FEM programs:

- Temperature and heat flux at nodal points of the body
- Heat source at nodal points and elements
- Through the relation (1.5) convective boundary conditions with heat flux

$$\tilde{q}=h_f\left(T_S-T_b\right) \tag{1.6}$$

where:

h_f = film coefficient [W/(m^{2}K)]

T_S = temperature at the surface of the body

T_b = bulk temperature of the adjacent fluid

For an approximate numerical determination of the values of a function $T\equiv T\left(x,y,z,t\right)$ in a body using the FEM several ways of formulation can be used. For the sake of clarity of the formulation process we will use the variational formulation.

It can be proved that a function satisfying equation (1.1) with its boundary conditions is equivalent in a variational formulation to a function that minimizes the integral (functional)

$$\Pi(T)=\frac{1}{2}\int_V\left[\lambda_x\left(\frac{\partial T}{\partial x}\right)^2+\lambda_y\left(\frac{\partial T}{\partial y}\right)^2+\lambda_z\left(\frac{\partial T}{\partial z}\right)^2-2\left(Q-\rho c\frac{\partial T}{\partial t}\right)T\right]dV+\int_{S_2}\tilde{q}TdS+\frac{1}{2}\int_{S_3}h_f\left(T_S-T_b\right)^2dS \qquad (1.7)$$

where S_2 is the area with the specified heat flux $\tilde{q}$ and S_3 is the area with convective heat transfer. *V* is the volume of the body.

The variational formulation of the non-stationary heat transfer problem for the body then reads as follows: It is necessary to find such a function, that satisfies the initial condition (1.3) and the boundary conditions (1.4), (1.5), that the integral (1.7) acquires a minimum value. Such a function exactly expresses the magnitude of temperatures at time *t* in points (*x*, *y*, *z*) of the body.

1.2 Geometric and temporal discretization of the problem. Element and body matrices

According to the same principles as at the structural problem we divide the body into finite elements and mark the nodal points. Let the total number of elements of the FEM model is BE and let the total number of the nodal points on the elemnt is EP. A function that approximates at time *t* the distribution of temperature in the e-th element from its values at the nodal points will be

$$T^e(x,y,z,t)=\left[N_1\left(x,y,z\right),N_2,\cdots,N_i,\cdots,N_{EP}\right]\begin{bmatrix}T_1(t)\\T_2\\\vdots\\T_i\\\vdots\\T_{EP}\end{bmatrix}=\mathbf{N}_e\mathbf{T}^e \qquad (1.8)$$

where the column matrix $\mathbf{T}^e(t)$ contains the element node temperatures and $\mathbf{N}_e(x,y,z)$ is the row matrix of the interpolation (shape) functions.

The integral (1.7) can be expressed as the sum of its values on the elements

$$\Pi=\sum_{e=1}^{BE}\Pi_e\left(T^e\right) \qquad (1.9)$$

where according to (1.7) is

$$\Pi_e=\frac{1}{2}\int_{V_e}\left\{\left[\lambda_x^e,\lambda_y^e,\lambda_z^e\right]\begin{bmatrix}\left(\frac{\partial T^e}{\partial x}\right)^2\\\left(\frac{\partial T^e}{\partial y}\right)^2\\\left(\frac{\partial T^e}{\partial z}\right)^2\end{bmatrix}-2\left(Q_e-\rho_e c_e\frac{\partial T^e}{\partial t}\right)T^e\right\}dV+\int_{S_2^e}\tilde{q}_eT^edS+\frac{1}{2}\int_{S_3^e}h_e\left(T_S^e-T_b\right)^2dS \qquad (1.10)$$

The condition of minimizing the functional for the computational model of the body can now be written as follows:

$$\frac{\partial\Pi}{\partial T_i}=\sum_{e=1}^{BE}\left(\frac{\partial\Pi_e}{\partial T_i}\right)=0 \qquad i = 1, 2, ..., EP \qquad (1.11)$$

For the derivative behind the sum sign in (1.11) we get

$$\frac{\partial \Pi_e}{\partial T_i} = \int_{V_e} \left\{ \left[\lambda_x^e, \lambda_y^e, \lambda_z^e \right] \begin{bmatrix} \frac{\partial T^e}{\partial x} \frac{\partial}{\partial T_i} \left(\frac{\partial T^e}{\partial x} \right) \\ \frac{\partial T^e}{\partial y} \frac{\partial}{\partial T_i} \left(\frac{\partial T^e}{\partial y} \right) \\ \frac{\partial T^e}{\partial z} \frac{\partial}{\partial T_i} \left(\frac{\partial T^e}{\partial z} \right) \end{bmatrix} - \left(Q_e - \rho_e c_e \frac{\partial T^e}{\partial t} \right) \frac{\partial T^e}{\partial T_i} \right\} dV + \int_{S_2^e} \tilde{q}_e \frac{\partial T^e}{\partial T_i} dS + \int_{S_3^e} h_e \left(T^e - T_b \right) \frac{\partial T^e}{\partial T_i} dS \tag{1.12}$$

Geometric discretization of this relationship is obtained when we introduce an approximation function for temperature (1.8) with discrete values of the temperature function on the element – the temperatures at the nodal points.

Then the derivatives of the functions appearing in (1.12) are

$$\begin{aligned} \frac{\partial T^e}{\partial x} &= \begin{bmatrix} \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial x} & \cdots & \frac{\partial N_i}{\partial x} & \cdots & \frac{\partial N_{EP}}{\partial x} \end{bmatrix} \mathbf{T}^e \\ \frac{\partial T^e}{\partial y} &= \begin{bmatrix} \frac{\partial N_1}{\partial y} & \frac{\partial N_2}{\partial y} & \cdots & \frac{\partial N_i}{\partial y} & \cdots & \frac{\partial N_{EP}}{\partial y} \end{bmatrix} \mathbf{T}^e \\ \frac{\partial T^e}{\partial z} &= \begin{bmatrix} \frac{\partial N_1}{\partial z} & \frac{\partial N_2}{\partial z} & \cdots & \frac{\partial N_i}{\partial z} & \cdots & \frac{\partial N_{EP}}{\partial z} \end{bmatrix} \mathbf{T}^e \end{aligned} \tag{1.13}$$

$$\frac{\partial}{\partial T_i} \left(\frac{\partial T^e}{\partial x} \right) = \frac{\partial N_i}{\partial x}; \quad \frac{\partial}{\partial T_i} \left(\frac{\partial T^e}{\partial y} \right) = \frac{\partial N_i}{\partial y}; \quad \frac{\partial}{\partial T_i} \left(\frac{\partial T^e}{\partial z} \right) = \frac{\partial N_i}{\partial z}; \quad \frac{\partial T^e}{\partial T_i} = N_i \quad \text{for } i = 1, 2, \ldots, EP \tag{1.14}$$

$$\frac{\partial T^e}{\partial t} = \mathbf{N}_e \begin{bmatrix} \frac{\partial T_1}{\partial t} \\ \frac{\partial T_2}{\partial t} \\ \vdots \\ \frac{\partial T_{EP}}{\partial t} \end{bmatrix} = \mathbf{N}_e \frac{\partial \mathbf{T}^e}{\partial t} \tag{1.15}$$

Note that in the relation (1.15) new unknowns appeared – derivatives of the nodal temperatures – written in the vector $\partial \mathbf{T}_e / \partial t$. This is a consequence of the fact that we are dealing with a time dependent problem.

Now we have already everything prepared for the relation (1.11) to express a set of equations for the e-th element according to (1.12)

$$\frac{\partial \Pi_e}{\partial T_i} = 0 \qquad i = 1, 2, \ldots, i, \ldots, EP \tag{1.16}$$

which provides matrices of the e-th element

$$\mathbf{C}^e \frac{\partial \mathbf{T}^e}{\partial t} + \mathbf{K}^e \mathbf{T}^e = \mathbf{f}^e \tag{1.17}$$

Matrix

$$\mathbf{K}_e = \int_{V_e} \mathbf{B}^T \mathbf{D} \mathbf{B}\, dV + \int_{S_3^e} h \mathbf{N}_e^T \mathbf{N}_e dS \tag{1.18}$$

containing

$$\mathbf{B}_e = \begin{bmatrix} \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial x} & \cdots & \frac{\partial N_i}{\partial x} & \cdots & \frac{\partial N_{EP}}{\partial x} \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_2}{\partial y} & \cdots & \frac{\partial N_i}{\partial y} & \cdots & \frac{\partial N_{EP}}{\partial y} \\ \frac{\partial N_1}{\partial z} & \frac{\partial N_2}{\partial z} & \cdots & \frac{\partial N_i}{\partial z} & \cdots & \frac{\partial N_{EP}}{\partial z} \end{bmatrix}; \qquad \mathbf{D}_e = \begin{bmatrix} \lambda_x^e & 0 & 0 \\ 0 & \lambda_y^e & 0 \\ 0 & 0 & \lambda_z^e \end{bmatrix}$$

is the conductivity matrix of the element and

$$\mathbf{C}^e = \int_{V_e} \rho_e c_e \mathbf{N}_e^T \mathbf{N}_e dV \tag{1.20}$$

is the specific heat capacity matrix of the element.

The "load" vector of the element is

$$\mathbf{f}^e = \int_{V_e} Q \mathbf{N}_e^T dV - \int_{S_2} \tilde{q} \mathbf{N}_e^T dS + \int_{S_3} h T_b \mathbf{N}_e^T dS \tag{1.21}$$

where its three terms represent the vectors of heat generation, heat flux and convection.

Ordered summation of element matrices (1.17) will yield to resulting set of differential equations with global matrices

$$\mathbf{C} \frac{\partial \mathbf{T}}{\partial t} + \mathbf{K}\mathbf{T} = \mathbf{f}(t) \tag{1.22}$$

The names of global matrices are analogous to the names of element matrices and the number of the equations in (1.22) is equal to the total number of nodes on the body FEM model.

In the case of a stationary problem, the set of equations (1.22) is simplified to a set of ordinary equations with a global vector of unknowns $\mathbf{T}$, which represents the temperatures at the nodal points of the model

$$\mathbf{K}\mathbf{T} = \mathbf{f} \tag{1.23}$$

As may be seen from the semi-discrete form of equation (1.22), the differential operator involving the time-dependent term still remains to be discretized and thus prepare the system for solution in time steps within the total time interval of the problem.

In ANSYS, the searched values of non-stationary problems are quantified in the time steps iterval

< 0, t_1, t_2, ..., t_n , t_{n+1} , ..., t_{max} > according to the built-in integration procedure. The principle of such a procedure can be most easily illustrated by using a simple difference method. By linearizing the time-dependent term in the (1.22) across time step $\Delta t = t_{n+1} - t_n$

$$\frac{\partial \mathbf{T}}{\partial t} \approx \frac{\mathbf{T}_{n+1} - \mathbf{T}_n}{\Delta t}$$

and by averaging the temperature values and vector values of the right side of the system, we get

$$\mathbf{C} \frac{\mathbf{T}_{n+1} - \mathbf{T}_n}{\Delta t} + \mathbf{K} \frac{\mathbf{T}_{n+1} + \mathbf{T}_n}{2} = \frac{\mathbf{f}_{n+1} + \mathbf{f}_n}{2}$$

From this, the resulting relation for determining temperatures at the nodal points of the model for individual time steps represents a set of ordinary equations

$$\left(\mathbf{C}+\frac{\Delta t}{2}\mathbf{K}\right)\mathbf{T}_{n+1}=\left(\mathbf{C}-\frac{\Delta t}{2}\mathbf{K}\right)\mathbf{T}_{n}+\frac{\Delta t}{2}\left(\mathbf{f}_{n+1}+\mathbf{f}_{n}\right)$$

which the program solves in a cycle through all time steps by analogy as a dynamic problem.

In the search for a steady temperature field, a set of ordinary equations is solved using the same methods as in the static strength problem.

After calculating the nodal temperatures of the body (vector $\mathbf{T}$), the program determines the approximation functions of the temperature in the elements and also the unknowns derived from them using the interpolation relation (1.8) through all the elements of the body. The results can then be analysed in the post-processor of the program and processed textually or graphically using the same procedures and means as for strength problems.

2. Radiation

Radiation is the propagation of any kind of electromagnetic waves in space. Its source is a permanent change in the electromagnetic fields of oscillating electrically charged particles of atoms. Electro-magnetic waves travel at the speed of light *c* and differ from each other in wavelength λ and frequency *f* whereby the relationship $f = c / \lambda$ holds. Their frequency spectrum is wide (Fig. 1), while thermal radiation is caused mainly by infrared rays with a wavelength from $\lambda = 0{,}8.10^{-6}$ to $0{,}1.10^{-3}$ m (0,8 to 100 μm). Any body that has a temperature higher than 0°K emits thermal radiation. When its temperature rises above body temperature (approx. 300 °K) at a certain distance we begin to perceive thermal radiation as heat radiation, and at high temperatures (e.g. a filament of a light bulb) thermal radiation also passes into the visible spectrum.

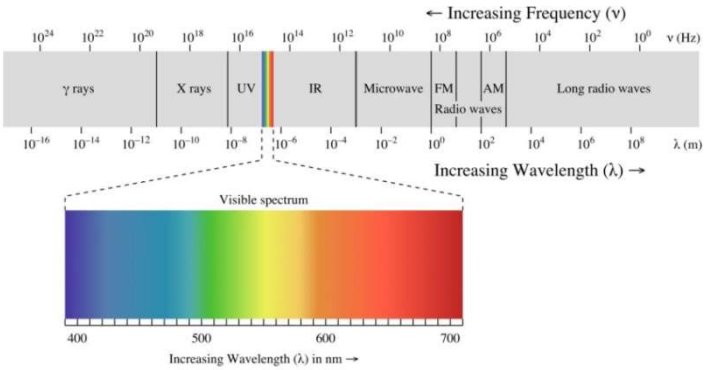


Fig. 2 *Frequency spectrum of electromagnetic waves*

When thermal radiation hits the surface of the body part of the energy is absorbed by the body, part is reflected and part is transmitted (Fig. 3).

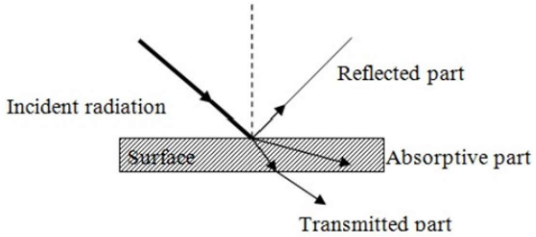


Fig. 3 *Effect of thermal radiation falling on a body*

The properties of the surface of the body on which the thermal radiation falls are characterized by the following dimensionless coefficients:

$\alpha =$ absorptivity - the ratio of absorbed heat energy to the total incident on the body
$\rho =$ reflectivity - the ratio of reflected thermal energy to the total incident energy on the body
$\tau =$ transmissivity - the ratio of the heat energy passed to the total incident on the body

The energy balance shows that the sum of these coefficients must be equal to one

$$\alpha + \rho + \tau = 1 \tag{2.1}$$

The energy that is reflected can be mirror reflected (the angle of incidence is equal to the angle of reflection) or diffuse (scattered, propagated in all directions).

Most solids and liquids are practically impermeable to heat flow ($\tau = 0$).

The above characteristic of the properties of the body and its surface on which thermal radiation falls can be simplified by introducing the concept of an ideal thermal emitter - a *thermal absolute black body* with the following basic properties:

1. The absolute black body absorbs all the incident thermal energy ($\alpha = 1, \rho = 0, \tau = 0$)
2. The density of radiated thermal energy (heat output per unit area) according to the Stefan-Boltzmann law is

$$q_0 = \sigma T^4 \qquad [\mathrm{W/m^2}] \tag{2.2}$$

where $\sigma = 5{,}67 \cdot 10^{-8}$ $\mathrm{Wm^{-2}K^{-4}}$ is the Stefan-Boltzmann constant and *T* is the absolute surface temperature of the body.

The absolute black body in thermal equilibrium with its surroundings absorbs all incident radiation from its surroundings (thermal radiation of all wavelengths) and emits the same amount of energy into space. An absolute black body can be imagined (and physically realized) as a hollow body with perfectly reflective walls with a small opening. If directed radiation enters the cavity only through this opening, it transfers all its energy to the body after multiple reflections from its walls. If the walls have a temperature of T, the radiation that comes out of the heated walls into the space is called the radiation of an absolute black body.

The simplest transfer of radiant thermal energy can then be imagined between the walls of two perfectly black bodies with temperatures $T_1 > T_2$. Let the radiating walls of the same size be parallel, extremely large and with a small gap between them (so that no part of the diffuse radiation is emitted outside the surfaces). Then the net value of the energy radiated from body 1 to body 2 is

$$q_{1\to 2} = \sigma(T_1^4 - T_2^4) \qquad [\mathrm{W/m^2}] \tag{2.3}$$

Real bodies emit thermal energy less efficiently than the absolute black body. In the state of thermodynamic equilibrium the temperature and therefore the internal energy of the body do not change. As much energy as it absorbs, it must also radiate. Therefore, the absolute black body emits but also absorbs the most energy compared to real bodies that have the same temperature. The ratio of the radiation intensities of a real body (at all wavelengths of thermal radiation) to the absolute black at the same temperature is called emissivity

$$\varepsilon = \frac{q}{q_0} \qquad [-] \tag{2.4}$$

For real bodies, the emissivity is greater than zero and less than 1. Its value is determined by the radiant properties of the surface and is dependent on temperature. In Table 1, we have listed the approximate emissivity values of some materials. Most metals are characterized by relatively low thermal emissivity unless they are oxidized. As a rule, non-metallic materials have a high emissivity.

Tab. 1 *Indicative emissivity values of certain materials*

Material	Emissivity
concrete - rough	0,94
wood	0,85
aluminium – polished	0,05
Aluminium – oxidised	0,25
cast iron	0,20
ice	0,96 – 0,98
cast iron – oxidized	0,60 – 0,90
steel – polished	0,08
Oxidized sheet steel	0,80
plastic – polypropylene	0,97
human skin	0,98
glass	0,92
snow	0,85
water	0,98

In commercial FEM programs, thermal radiation calculations are often limited to so-called *gray diffusion surfaces.* With such surfaces, both emissivity and absorption are independent of the wavelength of radiation, and the surfaces have a diffuse character with hemispherical radiation of a plane surface with cosine intensity. In a state of thermodynamic equilibrium based on Kirchhoff's law, the absorption of gray bodies is equal to its emissivity ($\alpha = \varepsilon$)and therefore materials that radiate heat well, at the same time absorb it well. Then, for two gray large parallel faces, the equation (2.3) can be adjusted to

$$\begin{aligned} q_{1\to 2} &= \varepsilon_1 \sigma T_1^4 - \alpha_1(T_1, T_2)\sigma T_2^4 \\ &\cong \varepsilon_1 \sigma T_1^4 - \varepsilon_1 \sigma T_2^4 \\ &= \varepsilon_1 \sigma (T_1^4 - T_2^4) \end{aligned} \tag{2.5}$$

If the radiating area of the body 1 is S_1 then its net radiating power is

$$Q_{1\to 2} = S_1 q_1 = S_1 \varepsilon_1 \sigma (T_1^4 - T_2^4) \quad [\mathrm{W}] \tag{2.6}$$

Consider the exchange of radiative thermal energy between the two gray body areas schematically shown in Fig. 4 and determine the magnitude of the heat flux transferred by radiation from area 1 to area 2.

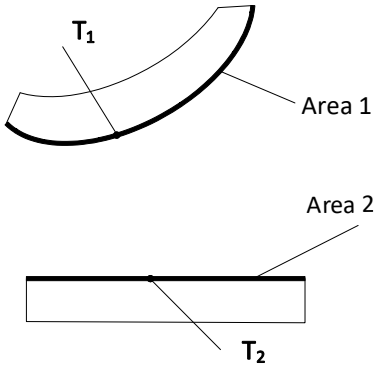


Fig. 4

Surface 1 radiates heat output in all directions and only part of it falls on area 2

$$Q_{1\to 2} = F_{1\to 2} S_1 \varepsilon_1 \sigma (T_1^4 - T_2^4) \tag{2.7}$$

where the coefficient $F_{1\to2}$, the so-called the *configuration factor* (which we need to know or calculate from the the size, shape and position of both surfaces), indicates how much of the total radiation output of surface 1 falls on area 2.

If the surfaces of the bodies reflect heat with reflectances ρ_1 and ρ_2, the solution is more complicated. According to (1) with $\tau = 0$ holds $\rho_1 = 1-\varepsilon_1$, $\rho_2 = 1-\varepsilon_2$ holds, and

$$Q_{1\to2} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1-\varepsilon_1}{\varepsilon_1 S_1} + \frac{1}{S_1 F_{1\to2}} + \frac{1-\varepsilon_2}{\varepsilon_2 S_2}} \tag{2.8}$$

These procedures and relationships can be generalized for the exchange of radiant heat between the surfaces of several bodies (see e.g. the ANSYS theoretical manual) and numerically solve such strongly nonlinear thermal radiation problems associated with other ways of transferring thermal energy. It remains to be briefly clarified the issue of determining factors $F_{i\to j}$ (automatically executed in the calculation program).

Configuration factors

The transfer of heat by radiation depends not only on the temperature of the bodies and their radiative properties, but also on their orientation relative to each other. This influence is expressed by a dimensionless coefficient called the configuration factor (slope factor, view factor), which can be defined as follows

$$F_{i\to j} = \frac{\text{radiant energy incident on a surface } j}{\text{total energy radiated by the surface } i}$$

It is assumed that the surfaces are isothermal, the radiation is diffuse, and the space between the emitting surfaces is a vacuum or gas that does not affect the exchange of radiative energy. The configuration factor in this case is a purely geometric quantity and can also be defined as a coefficient expressing how much of the view of the surface *i* is shaded by the area *j*. Some values of this factor can then be determined directly from such a definition (Fig. 5). It also follows that if area 1 is inside a fully enclosed area 2, then $F_{1\to2} = 1$.

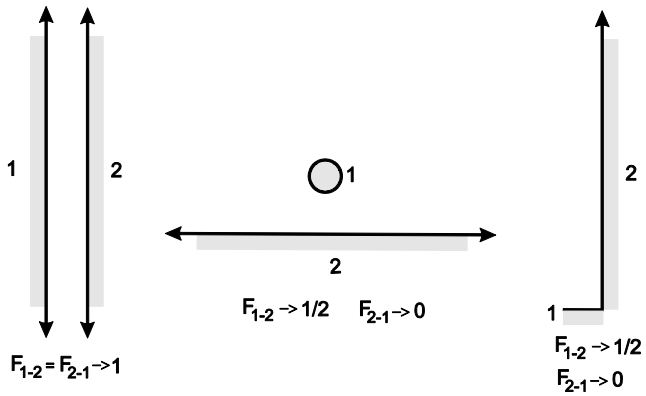


Fig. 5 *Values of configuration factors determined directly from the definition (two large parallel planes close to each other, spherical area close to a large planar face, small area perpendicular to large*

Other rules also make it easier to calculate configuration factors. One of them is the *rule of reciprocity*, which results from the energy balance of both bodies radiating to each other

$$S_i F_{i\to j} = S_j F_{j\to i} \tag{2.9}$$

Furthermore, it is a *summation rule* that applies to an energy-closed system. This is because if an area in an energetically closed system emits a heat flux Q_i on *n* surfaces, then the sum of the heat fluxes incident on

the individual surfaces of the system must be equal to this flux.

$$Q_{i\to 1} + Q_{i\to 2} + \ldots + Q_{i\to n} = Q_i$$
$$\frac{Q_{i\to 1}}{Q_i} + \frac{Q_{i\to 2}}{Q_i} + \ldots + \frac{Q_{i\to n}}{Q_i} = 1$$

And so we get the summation rule

$$F_{i\to 1} + F_{i\to 2} + \ldots + F_{i\to n} = 1 \tag{2.10}$$

In the summation rule, a non-zero value $F_{i\to i}$ applies only in the special case of surface concavity, as shown in Fig. 6

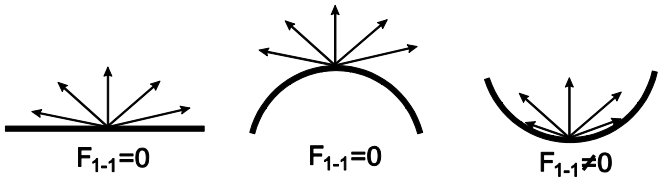
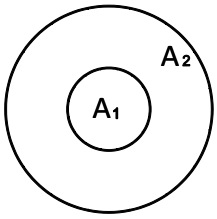


Fig. 6 *In a special case, the surface can also radiate to itself*

Example of using the above rules:

Determine the configuration factors for two concentric spherical surfaces A_1 and A_2:



According to the summation rule $F_{11} + F_{12} = 1$. Area 1 is convex and so we get

$$F_{11} = 0$$
$$F_{12} = 1$$

For area 2, the summation rule gives $F_{21} + F_{22} = 1$ and the rule of reciprocity applies $A_1 F_{1\to 2} = A_2 F_{2\to 1}$, so we get

$$F_{2\to 1} = A_1 / A_2$$
$$F_{2\to 2} = 1 - A_1 / A_2$$

The calculation of configuration factors for two general surfaces is more complicated. Using the definition and Lambert's cosine law a general relationship can be derived

$$F_{i\to j} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos\beta_i \cos\beta_j}{\pi r^2} dA_i dA_j \qquad [-] \tag{2.11}$$

where β_i and β_j are the angles that form the normals of the differential elements of the surfaces with their line (Fig. 7). The principle of reciprocity also applies to general areas.

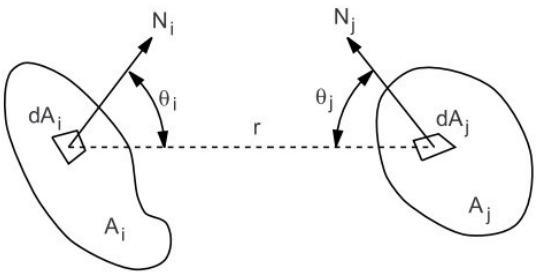


Fig. 7 *Figure for the calculation of the configuration factor of general areas*

The calculation of the configuration factor according to (2.11) leads to complex surface integrals even for relatively simple surfaces, which forced the creation of various formula or graphical catalogs for easier determination of configuration factors for frequently occurring combinations of surfaces. If we find the necessary combination in such a catalog, then the configuration factor can be easily determined from a simple algebraic formula. E.g. for two parallel concentric circular surfaces spaced apart by *a* (Fig. 7), the calculation formula is

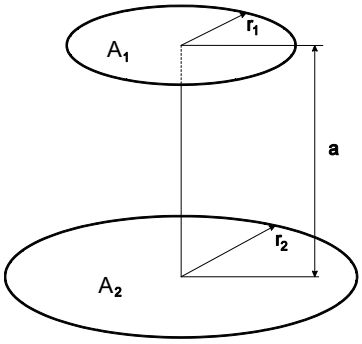


Fig. 7

$$F_{12} = \frac{1}{2}\left\{X - \left[X^2 - 4\left(R_2 / R_1\right)^2\right]^{1/2}\right\}$$

where $R_i = r_i / a$ and $X = 1 + (1 + R_2^2) / R_1^2$.

Calculation of heat transfer by radiation in ANSYS

For generalized radiation problems involving two or more surfaces two methods are involving in the ANSYS program: Radiosity solver method and Radiation matrix method. For simpler tasks, special elements can also be used to deal with radiation between points and between point and surface. The effective radiating surface area, the form factor and emissivity can be specified as real constants for each radiating point.

In these methods, all the simplistic assumptions that we considered in the previous sections are introduced. This is primarily the assumption of grey and diffuse properties of the emitting surfaces and that the radiation is not affected by the space between the radiating surfaces. (Some of these limitations can be circumvented in ANSYS FLUENT.) The problem of heat exchange with radiation is always solved iteratively (by the Newton-Raphson method), because the temperature changes on the surfaces that participate in the heat exchange by radiation. The iteration process is completed when an energy equilibrium is achieved between the bodies within a certain prescribed tolerance.

When solving stationary problems, it is necessary to realize that we are looking for the so-called steady state (unchanging in time). Let's imagine two bodies with different temperatures, at which heat transfer occurs only by radiation. If we do not supply energy to the warmer body and take it from the colder one, then the temperatures of the bodies will equalize and the heat flow in the steady state between the two bodies will be zero. This means that in the steady state, non-zero heat exchange occurs only when conductive and/or convective boundary conditions also occur on the bodies.